Robust Solar Hosting Capacity for Pasadena's Decarbonization

A Mathematical Program for Robustness Against Uncertainty

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IN JANUARY 2023, THE CITY OF PASADENA, CA, USA, set a goal to provide 100% of the city's electricity from carbon-free resources by 2030. In this article, we describe some challenges in the path toward 100% carbon-free electricity in Pasadena and potential mitigation strategies. Through an ongoing collaboration between the city's municipal utility and Caltech, these real-world considerations are formalized as system models and optimization programs. In particular, we focus on the planning of distributed solar photovoltaic (PV) power in the Pasadena distribution system as one step toward the decarbonization goal. We

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discuss the following questions: 1) What are the grid impacts from widespread and distributed PV adoption? 2) What is the maximum PV hosting capacity while ensuring safe operations? 3) What are potential infrastructure upgrades to mitigate the impact of large-scale PV adoption?

Pasadena Distribution Grid

Pasadena has a population of approximately 140,000. Its municipal utility, Pasadena Water and Power (PWP), serves more than 67,500 residential and commercial customers in a 23-square-mile service area with a budget of US$333 million in the 2024 fiscal year. The city council of Pasadena set a policy goal in January 2023 to source 100% of its electricity from carbon-free generation by 2030. To appreciate the challenges in achieving this goal, we start by summarizing some facts on the current system. See the section "For Further Reading" for a detailed public report on the Pasadena system.

Current System

PWP has been operating its own power plant (Glenarm) for more than 120 years. The Glenarm plant had five gas and combined cycle units with a total nameplate capacity of approximately 225 MW in 2022. Today, however, the large majority of Pasadena's electricity is imported from the California Independent System Operator (CAISO) grid at its Goodrich receiving station in East Pasadena.

The Goodrich receiving station imports power over 220-kV transmission lines from two Southern California Edison (SCE) substations using three 220-kV/34.4-kV transformers. The import capacity is 280 MW, out of a 336-MW contracted limit, to account for $N-1$ contingencies (Pasadena Water and Power (PWP) 2022). Power from the receiving stations at Goodrich and Glenarm is transported over a 35-kV subtransmission system to 10 of its 11 distribution substations, with one of the 11 distribution substations served by 17-kV lines [Figure 1(a) and (b)]. The 10 distribution substations transform the 35-kV subtransmission voltage to either 17 kV or 4 kV to be distributed to its customers on distribution feeders throughout the city. These 11 distribution substations have a total of 26 transformers and supply 109 distribution feeders at voltages of 17 or 4 kV. Reactive power is dispatched from the Glenarm power plant and capacitor banks and imported from the CAISO market to maintain voltages on the system. Bulk power battery and phase-shifting transformers are also being deployed to manage reactive power demand.

The Pasadena grid consists of almost 1,000 miles of distribution and subtransmission lines. The system peak was 330 MW in 2024. The annual energy delivered by PWP is around 1,000 GWh and decreasing due to behind-the-meter distributed energy resources (DERs), such as residential PV and fuel cells as well as energy efficiency programs (Figure 2). Going forward, the net load is expected to increase because of electric vehicle (EV) adoption and building electrification.

Decarbonization Challenges

Accomplishing the goal of decarbonizing Pasadena's electricity by 2030 requires significant changes to the procurement of generation sources, infrastructure upgrades, and sustained effort to increase local DERs. However, many challenges remain.

1) The majority of electricity in Pasadena will continue to be imported at the Goodrich receiving station on the east of the city, and a large portion will be transported to the west of the city. Both the import capacity at Goodrich and the capacity of the major east–west connection will need to be increased to accommodate the expected load growth (see Figure 1).
2) There are limitations in the current system capacity to support large-scale DERs adoption, particularly from bus voltage and line violations.
3) There is a lack of local energy storage to meet 24/7/365 carbon-free energy.
4) There is a lack of ability to control and coordinate DERs as well as limited system visibility (data availability) at voltage levels below 4 kV.

In response, PWP is pursuing a set of tasks, including the following:

1) Major capital projects to accommodate load growth and enable the carbon-free transition by upgrading aging infrastructure. For example (Pasadena Water and Power (PWP) 2022), PWP may increase the import capacity of the Goodrich receiving station from 280 to 336 MW by replacing the three existing transformers with larger transformers and upgrading the 220-kV system's protection schemes. PWP may also install equipment to reduce short circuit currents on its 35-kV system to under the 35-kV circuit breaker rating (40 kA), so that PWP can freely dispatch all of its five generating units at Glenarm in the event that the power import at the Goodrich

receiving station becomes unavailable. Some of the 35-kV subtransmission cables and 4-kV substation units may also be replaced to better support power delivery across the city during peak demand, especially on the east–west connection from the Santa Anita receiving station to the Glenarm receiving station, which currently carries around half of the imported power from the CAISO grid to the west side of the city [Figure 1(b)].

2) Procurement of renewable generation sources, such as wind, solar, and geothermal power, through long-term contracts.

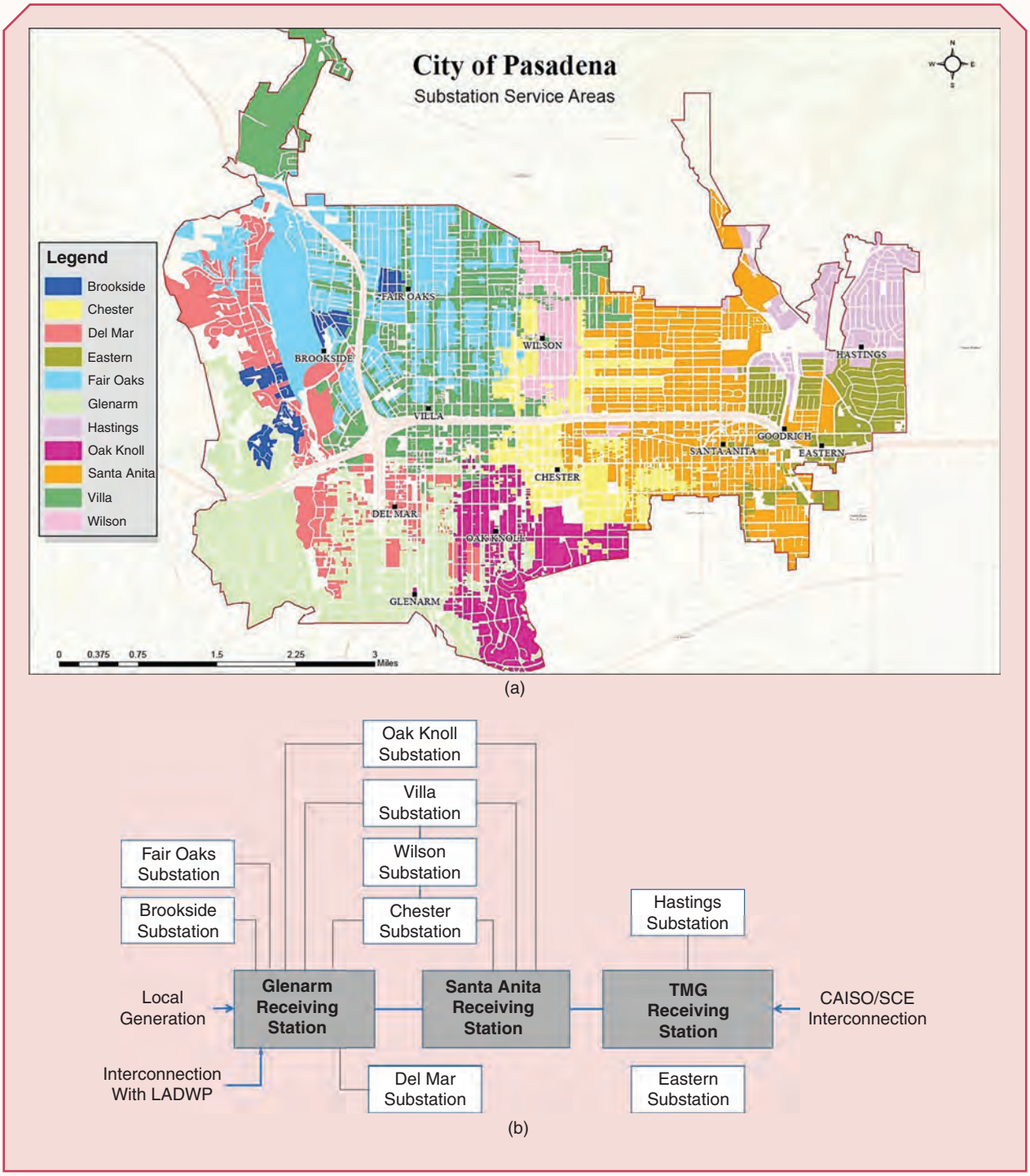

figure 1. (a) PWP substation service areas. (b) PWP subtransmission system. LADWP: Los Angeles Department of Water and Power. (Source: "Power delivery master plan," PWP, 2022.)

3) Planning for local energy storage to improve reliability and support 100% hourly demands with carbon-free resources, although it is unclear if any *long-duration* energy storage product will be mature enough and suitable for deployment in Pasadena by 2030.
4) Rollout of advanced metering infrastructure to increase low-voltage system visibility and allow for time-of-use rates.
5) The design of incentive programs for behind-the-meter DERs in Pasadena as well as direct deployment of PWP-owned-and-operated resources. They include PV panels, batteries, and also demand response programs. These DERs can reduce the amount of carbon-free electricity that must be imported as demand grows and increase availability during contingency events.

In this article, we focus on some planning and operational challenges in the last item, DER deployment. To make our discussion concrete, we use PV power adoption in Pasadena as an example to illustrate the challenges in increasing local renewable generation. We present the effects of different levels of PV adoption as well as methods for estimating the solar hosting capacity. Finally, we briefly discuss potential strategies to mitigate the impact of large-scale PV adoption. Even though our discussion focuses on PV adoption, these issues are relevant to all DER deployments, such as batteries, EVs, smart PV inverters, and demand response resources. These questions arise naturally: 1) What are the grid impacts from widespread PV adoption? 2) What is the maximum PV installed capacity without violating voltage and line limits? 3) What can be done to increase the hosting capacity? These questions are addressed in the sections "PV Adoption: Grid Impact" and "PV Adoption: Regulated Capacity."

PV Adoption: Grid Impact

The most relevant safety constraints in the Pasadena grid are (over)voltage at customer buses and (over) loading in transformers and lines. To quantify these effects, we perform Monte Carlo simulation using real load time series and solar generation profiles based on real historical weather data. We describe the system model in the next section and define appropriate performance metrics in the section "Performance Metrics" to quantify the effect of PV adoption at various penetration levels.

System Model

The variables of interest are the bus voltages and branch power flows, which must be kept within safe limits. These quantities are determined by the network topology and parameters as well as the net load at each customer bus.

Each customer's net load is the total demand minus local PV generation, if any. The PV generation depends on its installed capacity and its capacity factor, which depends on the weather, tilt angle, etc. Therefore, the installed PV capacities affect bus voltages and line flows, and we represent these relations by

$$V_i = f_i(\zeta), \qquad S_{ij} = g_{ij}(\zeta)$$

where V_i denotes the voltage magnitude at bus i, S_{ij} denotes the apparent power flow in line connecting buses (i, j), and ζ denotes the vector of installed PV capacities at every bus. If a bus has no PV power, its capacity ζ_i is zero. We call the vector ζ the PV installation profile for the entire circuit. The functional form of f_i, g_{ij} depends on the circuit (power flow) model. In this study, we use the three-phase extension of the Linear DistFlow model, which is suitable for unbalanced distribution systems.

It is important to note that the bus voltages V_i and the apparent powers S_{ij} are random because of randomness in the PV generation capacity

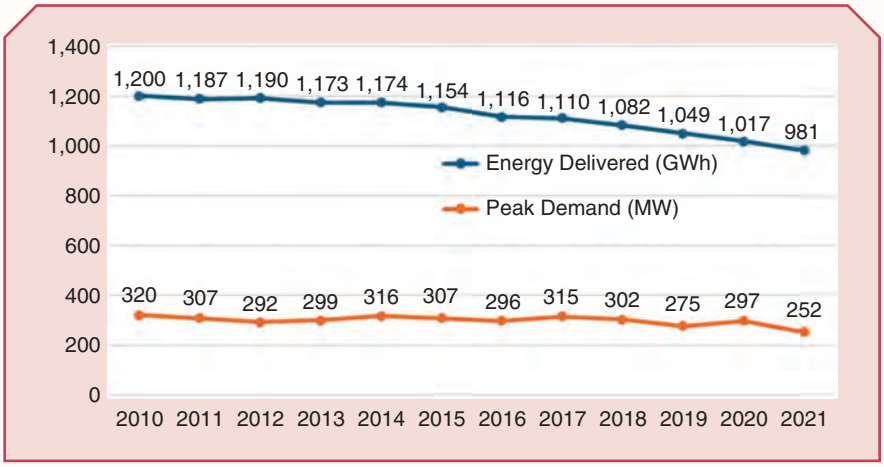

figure 2. Energy delivered and peak demand during the years 2010–2021 in Pasadena. (Source: "Power delivery master plan," PWP, 2022.)

factor and installation profile. We will illustrate how the PV installation profile ζ can cause voltages and line power to violate their limits.

Performance Metrics

Example Feeder

The numerical studies are performed on an example feeder from the PWP service territory (Figure 3). The test system has approximately 800 buses, and about half are connected to customers. All of the customer buses are potential locations for PV installation. The load injection profiles are based on actual historical measurements at the PWP feeder. The PV generation profiles are based on actual historical weather data in Pasadena from the National Renewable Energy Laboratory National Solar Radiation Database.

Grid Impacts

The system model in the section "System Model" reveals that the bus voltages V_i and branch power flow S_{ij} are functions of the installed PV capacity ζ. For safe operation, the voltages on the distribution feeder must be maintained to within 5%–10% of their nominal values V_i^{nom}, e.g., $0.95\,V_i^{\text{nom}} \le V_i \le 1.05\,V_i^{\text{nom}}$. Similarly, branch power flows (lines and transformers) must be below their rated capacities. We use two performance metrics to illustrate how the PV installation profile ζ can cause constraint violations.

The first metric is the overvoltage *violation probability* for the circuit:

$$\text{Prob}(V_i > 1.05V_i^{\text{nom}})$$

which depends on the PV installation profile ζ. This is the probability that *any* bus violates its voltage limit. As shown in Figure 4, the average overvoltage violation probability increases as the PV penetration level (i.e., $\Sigma_i \zeta_i$) increases. Different PV installation profiles ζ can have the same penetration level, and therefore each penetration level corresponds to a range of ζ and hence a range of violation probabilities.

Since each penetration level may correspond to different PV installation profiles across customers, several different installation profiles are sampled for each level to characterize the range of violation probabilities. The violation probability for each installation profile is calculated as the empirical probability from randomly sampled load and solar generation scenarios across multiple years.

The second metric illustrates the *spatiotemporal clustering* of overvoltage violations. Using the load and PV generation data on the example feeder, we evaluate the voltage violation probability as defined previously for each 5-min interval over the course of 24 h.

By running Monte Carlo simulations over multiple days, we obtain for every bus and 5-min interval an estimate of the probability that the voltage magnitude violates its upper limit. As shown in Figure 5, at 1.3-MW (43%) penetration, few buses violate voltage limits and violations occur around noon. Our proposed regulation policy, using a robust scenario-based formulation that considers installation and load/generation uncertainties, enables violation-free operation up to 1.3-MW

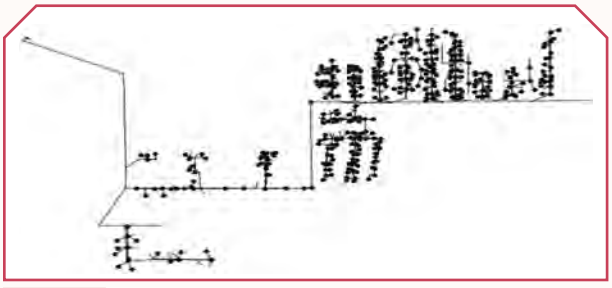

figure 3. Example feeder. Dots represent buses.

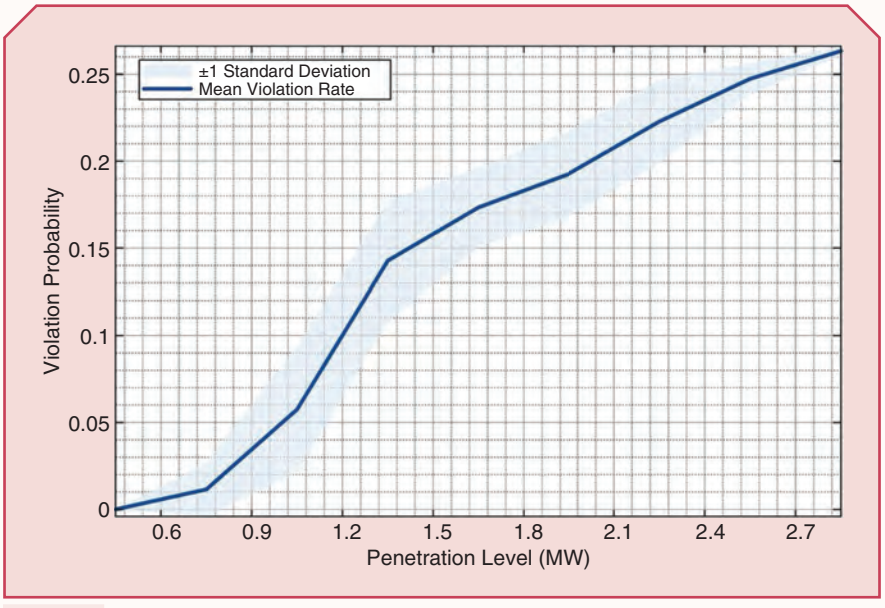

figure 4. Voltage violation probability versus penetration level.

penetration. At a 2.7-MW (90%) penetration level, most buses experience voltage violations throughout the day.

The preceding two performance metrics can be similarly defined for undervoltage violations, which are relevant to large-scale EV charging. They can also be similarly defined for line and transformer power capacity violation. The impact of PV installation capacity ζ online limit violations can be illustrated using the same metrics.

PV Adoption: Regulated Capacity

One way to avoid violations of voltage and line constraints is to limit the maximum PV capacity each customer can install. We call this amount *the regulated capacity* for the customer. The sum of regulated capacities for all customers is the hosting capacity of the overall system. We note that the term *hosting capacity* does not have a unique definition. Generally, the hosting capacity is the maximum solar generation capacity (in kilowatts) that a system can accommodate without violating its operation constraints. However, the quantity depends on the particular assumptions, the system model, constraint limits, uncertainty, and problem formulation.

In this section, we propose a method to determine the regulated capacities for customers so as to maximize the overall PV hosting capacity. The result should consider fairness and ensure safe operations. This requires a systematic quantification of the factors, including

- the load and PV generation profiles, which are uncertain
- the distribution grid topology, parameters, and operating constraints
- the capacity, location, and operation of energy storage and other controllable devices on the grid.

In the following, we briefly survey the existing approaches and precisely define the problem formulation in this case study.

Motivation and Existing Approaches

A major difficulty in hosting capacity estimation is uncertainties, both in PV adoption and in random fluctuations of PV generation and load. All of these uncertainties should be considered in the PV hosting capacity estimation to ensure safe operation. The relevant literature on PV hosting capacity is extensive; here, we briefly review existing approaches only to motivate our formulation.

Two primary approaches exist for estimating PV hosting capacity. The utility-based approach assumes that the utility centrally plans and determines PV installations within its service territory. By contrast, the customer-based approach assumes that the utility cannot directly control customers' PV installations; instead, it specifies a maximum allowable PV capacity per customer to maintain safe grid operation. While the utility-based approach minimizes uncertainty in PV installations, it is rarely used in practice. The customer-based approach, on the other hand, introduces substantial uncertainty in the resulting installed PV capacities.

To account for uncertainties, either simulation or optimization techniques are utilized. Simulation-based approaches typically rely on Monte Carlo sampling to generate random scenarios of load and PV generation profiles as well as customer PV capacities. The common procedure involves generating these scenarios, performing power flow analyses, evaluating key variables (e.g., voltage, current, and losses) against performance limits, and determining the hosting capacity accordingly. However, this process is computationally intensive as it requires

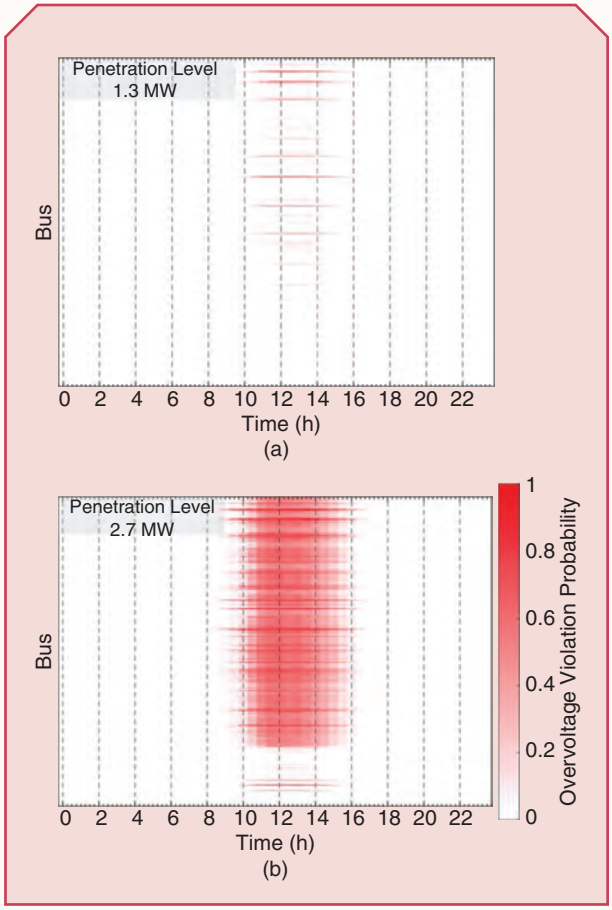

figure 5. Spatiotemporally varying overvoltage rate at PV penetration levels of (a) 1.3 MW and (b) 2.7 MW.

Generally, the hosting capacity is the maximum solar generation capacity (in kilowatts) that a system can accommodate without violating its operation constraints.

generating a large number of scenarios and running power flow simulations for each one. Furthermore, the number of scenarios needed to achieve desired risk and confidence levels is rarely analyzed rigorously and is usually chosen empirically. In addition, these methods generally lack guarantees of optimality.

Optimization-based methods formulate hosting capacity estimation as an optimization problem, where the objective is to maximize total PV capacity subject to system safety and operational constraints. Some studies adopt a deterministic formulation, ignoring uncertainties in customer adoption and load/generation variations. Others incorporate load and generation uncertainties using robust optimization. However, defining appropriate uncertainty sets can be challenging—overly large sets lead to conservative solutions, while overly small sets may fail to achieve the desired risk level. An alternative is scenario-based optimization, which addresses uncertainties by directly sampling scenarios from historical data or probabilistic models. In this context, some works employ conditional value-at-risk metrics to evaluate and manage the risk associated with load and generation uncertainties within the scenario-based framework. However, for the formulation to be computationally tractable, existing methods either neglect the uncertainty in customer installation or handle it through Monte Carlo sampling.

As a concrete example, we formulate the hosting capacity estimation problem as a robust-scenario program that incorporates uncertainties in both customer installation and load/generation profiles.

Specifically, the uncertainty in customer PV installation is incorporated in the robust formulation. The uncertainties in load/generation profiles are addressed via scenario programming so as to avoid constructing uncertainty sets and guarantee desired risk and confidence levels. A linearized unbalanced three-phase power flow model is used. We equivalently reformulate the robust scenario program (RSP) as a linear program. A solution of the linear program provides a maximum allowable PV capacity limit for each customer. This is the *hosting capacity*. As long as the customer-installed PV capacities do not exceed their regulated capacities, all voltages and loadings on transformers and lines are guaranteed to remain within their limits. The solution offers clear, implementable policy recommendations with quantifiable confidence levels.

PV Hosting Capacity Calculation

In this section, we discuss how we model uncertainty in customer PV installation and in load/generation profiles and formulate an RSP for PV hosting capacity estimation.

PV Hosting Capacity

Consider the example feeder in the section "Performance Metrics," where solar PV generation can potentially be installed on the customer buses. The utility cannot dictate directly how much PV capacity each customer must install. Instead, the utility can set a maximum allowable capacity limit $\bar{\zeta}_i$ for each customer to ensure safe operations. The limit is the *regulated capacity* for that customer. We define the *hosting capacity* of the system as the sum of the regulated capacities at all buses in the system, i.e., $\Sigma_i \bar{\zeta}_i$.

Uncertainties

There are two sources of uncertainty. First, the customers can choose PV installations *up to* the regulated capacities; i.e., customers can choose any $\zeta_i \leq \bar{\zeta}_i$. The hosting capacity $\bar{\zeta}_i$ therefore must be chosen robustly against all possible ζ_i. In other words, the system voltage and line limits must be satisfied for any customer installations within the allowed limit. As we will see next, this is handled via robust optimization.

The second type of uncertainty lies in the time-varying load and solar capacity factors, which depend on a myriad of factors, such as the weather and time of day. This is handled via scenario programming, which samples a set of time points for load and solar generation

Semi-infinite programs are generally intractable, but remarkably, in this case the RSP is equivalent to a linear program and hence is efficiently solvable.

$\{(p_{\text{load}}(t), q_{\text{load}}(t), \bar{p}_{\text{pv}}(t)) : t = 1, \ldots, T\}$ from historical data.

RSP

We formulate the problem of estimating the PV hosting capacity as the following RSP:

$$\max_{\bar{\zeta}_i} \sum_i \bar{\zeta}_i + F(\bar{\zeta})$$

$$\text{subject to } 0.95V_i^{\text{nom}} \le V_i(\zeta, t) \le 1.05V_i^{\text{nom}},$$
$$S_{ij}(\zeta, t) \le S^{\max}$$

$$0 \le \bar{\zeta}_i \le \bar{\zeta}_i^{\max}$$

$$\text{for all } \zeta_i \in [0, \bar{\zeta}_i],\ \forall i,\ \forall ij,\ \forall t \in \{1, \ldots, T\}.$$

This formulated problem has three simple structures.

First, the goal is to maximize the hosting capacity and a fairness measure $F(\bar{\zeta})$ over the regulated capacities $\bar{\zeta}$. The fairness measure $F(\bar{\zeta})$ ensures the regulated capacities $\bar{\zeta}_i$ are allocated equitably among the customers i. In the following case study, we will illustrate the spatial distribution of optimal regulated capacities with and without the fairness measure $F(\bar{\zeta})$.

Second, the regulated capacity $\bar{\zeta}_i$ of customer i must be chosen within some physical limit $\bar{\zeta}_i^{\max}$, such as its rooftop area. The customer can install any amount ζ_i up to its regulated capacity $\bar{\zeta}_i$.

Finally, the voltage magnitudes $V(t)$ and the line apparent powers $S(t)$ depend on the actual customer PV capacities ζ_i, and their upper bounds $\bar{\zeta}_i$ must be chosen so that the voltage magnitudes and branch power satisfy their limits for any PV installation profile ζ up to $\bar{\zeta}$ and for T random samples of load and generation profiles.

The RSP is called *semi-infinite* because it involves an uncountably infinite number of constraints on voltages and line flows indexed by $\zeta_c \in [0, \bar{\zeta}_c]$. Semi-infinite programs are generally intractable, but remarkably, in this case the RSP is equivalent to a linear program and hence is efficiently solvable. Finally, unlike Monte Carlo simulations, the sampling of scenarios comes with performance guarantees in terms of sample

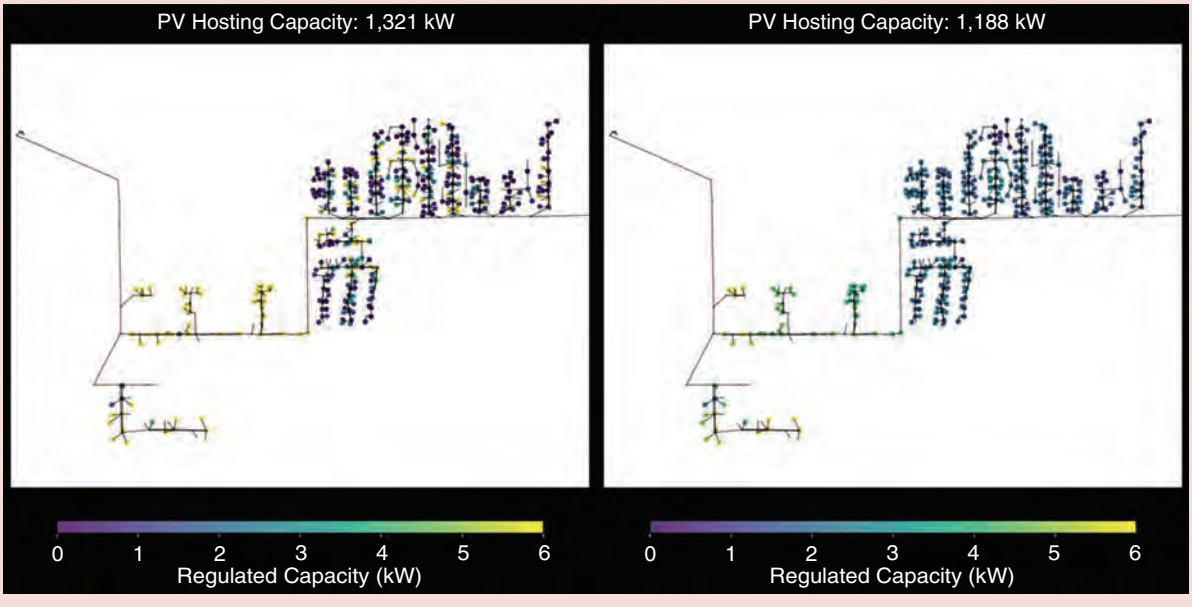

figure 6. Spatial distribution of regulated capacities (a) without and (b) with the fairness objective.

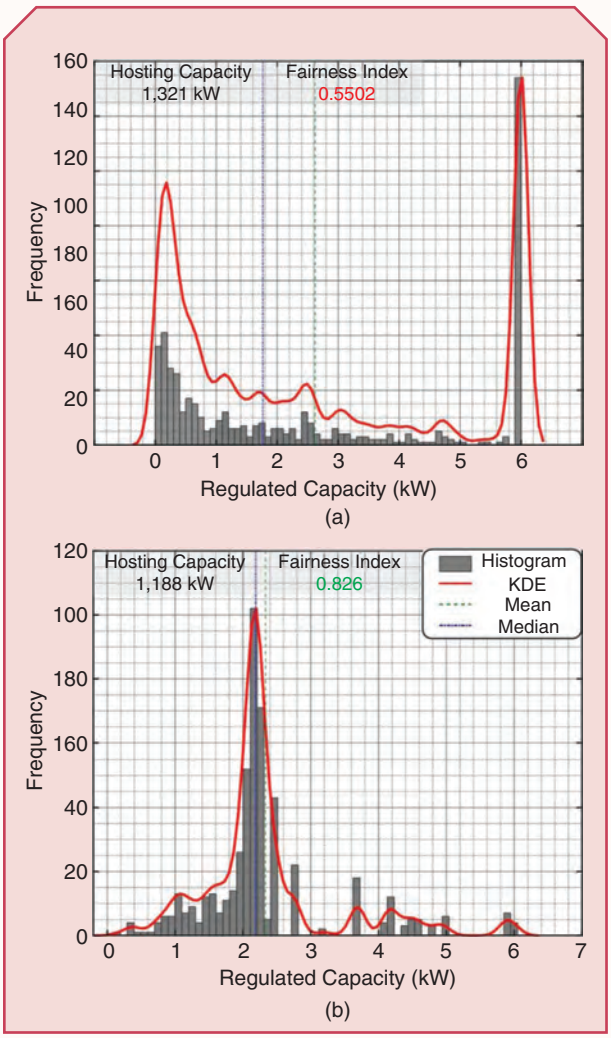

figure 7. Regulated capacities distribution (a) without and (b) with the fairness objective. KDE: kernel density estimation.

complexity and violation probabilities. More details will be included in a journal submission of this work.

Case Study

Solving the RSP on the example feeder yields the optimal regulated capacities $\bar{\zeta}_i$ for each customer, which are PV capacity limits that ensure safety. We first look at the distribution of regulated capacities of the feeder and then illustrate how to improve the fairness of capacity allocation among customers without significantly sacrificing the total hosting capacity.

As shown in Figure 6(a), the sum of regulated capacities is 1,321 kW without the fairness objective. According to Figure 4, this penetration level would have resulted in more than 10% voltage violation probability without regulated capacities. The regulation policy ensures that the system can operate safely at this level for the scenarios considered.

Figure 6(a) shows a spatial distribution of the optimal regulated capacities without the fairness measure $F(\bar{\zeta})$, which generally decreases further away from the feeder head. This is due to decreasing transmission capacity (branch limits) and increasing line impedance (voltage limits). Figure 7(a) shows the empirical distribution of regulated capacities without the fairness measure $F(\bar{\zeta})$: it is roughly a bimodal distribution where the majority of customers either have a high regulated capacity (6 kW) or less than 1 kW regulated capacity. To measure the fairness of $\bar{\zeta}$, we use Jain's fairness index, defined as

$$J(\bar{\zeta}) := \frac{\left(\sum_i \bar{\zeta}_i\right)^2}{n\sum_i \bar{\zeta}_i^2}.$$

Jain's index ranges from 0 to 1. The larger Jain's index, the fairer the allocation (e.g., $J(\bar{\zeta}) = 1$ if $\bar{\zeta}_i$ are all equal). For Figure 7(a), Jain's index for the feeder is 0.55.

By including the fairness measure $F(\bar{\zeta}) := \gamma \Sigma_i \log(\bar{\zeta}_i + 1)$ in the objective, the inequality in the optimal regulated capacities is reduced. Here γ is a positive weight to control the tradeoff between the hosting capacity $\Sigma_i \bar{\zeta}_i$ and fairness, and 1 is a small positive number to avoid taking the logarithm of 0. Figures 6(b) and 7(b) show the results with $F(\bar{\zeta})$. Jain's index has increased from 0.55 to 0.83, while the hosting capacity of the feeder decreased by 10%, from 1,321 to 1,188 kW.

Mitigation Strategies

Finally, we remark on different approaches to mitigate the grid impacts of PV adoption and increase the hosting capacity.

Infrastructure Upgrade

A common and traditional approach is to upgrade lines and transformers to a higher capacity to mitigate congestion and replace lines with a lower impedance to mitigate voltage deviation. However, doing so strategically is nontrivial.

For example, which line(s) should be upgraded to achieve the maximum increase in the system hosting capacity? The optimal dual variables from a solution of the preceding RSP guide our decision.

The optimal dual variables associated with each line constraint in the RSP are equal to the marginal change in the objective value with respect to a change in the line capacity limit. In other words, the optimal dual variable indicates how much additional PV hosting capacity can be gained by upgrading a line. A similar analysis can be performed for voltage constraints, albeit with a few additional steps since one cannot directly "upgrade" the voltage limit of a bus.

Centralized Planning

In some cases, the utility can directly control the amount of PV capacity installed. This reduces the uncertainty and increases the PV hosting capacity. For example, Pasadena can install and operate PV systems on city-owned buildings, parks, and land. This can be easily incorporated into the RSP formulated previously by directly optimizing over the PV capacity ζ_i, instead of its regulated capacity $\bar{\zeta}_i$, for buses i owned and operated by PWP.

DERs

PV panels and batteries are connected to the grid through inverters, which are power electronic devices that allow flexible control of active and reactive power within device limits. For example, a simple linear feedback control law for reactive power $q_i(t+1) = q_i(t) - k_i(v_i - v_i^{ref})$ can be implemented by the inverter at each bus i, where k_i is the control gain. The approach lies in a rich body of work on volt-var control. Such a control law guarantees that voltages converge to the reference value without requiring a communication infrastructure. Similarly, active power can also be controlled to support voltage regulation. These control laws can also be incorporated into the scenario program to characterize how smart inverter control capabilities could help increase the PV hosting capacity.

Conclusion

Pasadena's 2030 carbon-free electricity target requires not only ambitious policy commitments but also rigorous technical planning to ensure grid reliability and safety. Using PV adoption as a concrete example, we have demonstrated some of the challenges in DER deployment and methods for assessing and mitigating its grid impacts and estimating the hosting capacity. We have formulated a mathematical program to optimize PV hosting capacity that is robust against the uncertainty in customer PV installation and randomness in load/generation profiles. Strategies including infrastructure upgrade, centralized planning, and smart inverter control can be further implemented to increase the hosting capacity and mitigate effects brought about by large-scale PV integration. While our analysis focused on solar PV, the same technical considerations—voltage regulation, protection coordination, and optimal control—apply broadly to other DERs, such as energy storage, EVs, and demand response resources. We show that a mathematical approach with optimality and safety guarantees is possible. This approach provides a general framework for large-scale DER planning and integration for utilities and municipalities undertaking similar decarbonization goals.

For Further Reading

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